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LETTER TO THE EDITOR

Life–death transition in a random biological model

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Abstract. Species live in an environment where nutritious and poisonous food is randomly distributed. A survival criterion is analysed based on how poor on average the food can be.

In this letter we are interested in a particular type of birth–death process which may arise in some biological situations and, perhaps, chemical reaction systems. Suppose there are certain amounts of input species concentrated initially at the space origin; in space there are randomly distributed positive and negative stimulants which enable the species to proliferate or decrease at exponential rates, respectively. These stimulants can be, for example, good food and poor food, or positive and negative catalysts, and so on. Here we assume the idealised situation in which the stimulants themselves are not affected or consumed by the species' presence. This model can then be summarised by the following equation:

$$\dot{P}(x, t) = \Delta P(x, t) + V(x)P(x, t) \tag{1}$$

and

$$P(x, 0) = \delta(x)$$

where the first term in the RHS represents the diffusion and $V(x)$ assumes randomly positive and negative values and a short-ranged correlation function, i.e. it has a Gaussian distribution. $P(x, t)$ is the local population density at a given time. For a qualitative description it suffices to set all coefficients to unity in (1).

This model is a slight generalisation of the random trapping model (Kang and Redner 1985, Blumen *et al* 1983) in which $V(x)$ takes only random non-positive values. The above authors reached the conclusion that the total surviving species decreases as $\rho(t) \sim \exp(-t^{\alpha})$ where $\rho(t) = \int P(x, t)\, \mathrm{d}x$ and α is the so-called survival exponent of order one. In a recent study (Zhang 1986) it is shown that the relative distribution function is concentrated and the population centre hops in space as $x_c \sim (t/2 \ln t)$. Such a migration is possible because the species try to survive best by finding better and better shelters where good food is abundant (larger and larger $V(x)$ values).

Clearly, if on average $V(x)$ is positive, i.e. $\langle V(x) \rangle > 0$, the species will proliferate forever. A moment's thought will convince the reader that if $\langle V(x) \rangle = 0$ the species will also survive, finding some shelter (a place with positive $V(x)$) nearby via diffusion and proliferate there. An interesting question is that even when the environment is unfavourable, i.e. $\langle V(x) \rangle = -V_0 < 0$, can the species eventually survive? We would like to find the dividing value V_0 which separates survival and extinction. In this letter we will give an estimate of this value, and in the following analysis only the leading order is retained.

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The expression

$$\langle V(x)\rangle = -V_0 \qquad (2)$$

where V_0 is positive, indicates that there are more negative stimulants than positive ones. In order for the species to survive, we have to find a place where $V(x)$ is positive. The chance of finding such a place at a single site is proportional to

$$\int_0^\infty \exp[-(V+V_0)^2]\,\mathrm{d}V \sim \exp(-V_0^2). \qquad (3)$$

For a given time t the population centre can hop a distance $x_c(t) \sim t/(2\ln t)$. The surviving condition must be (total probability of finding a shelter)

$$x_c(t)\exp(-V_0^2) \sim \mathrm{O}(1) \qquad (4)$$

(for arbitrary spatial dimension the analysis is trivially extended). However, before reaching this first shelter, the species meet only random negative $V(x)$ as in a random trapping model, and we know that during this time period the total surviving species decrease exponentially as $\rho(t) \sim \exp(-t^\alpha)$. From a practical point of view there should be a lower limit of $\rho(t)$ below which the species should be regarded as extinct. For example, this cutoff can be at a single reproductive cell or a molecular level. Let us introduce an extremely small number ε as this cutoff, i.e. we must have $\rho(t) \geqslant \varepsilon$, otherwise the species should be pronounced dead with no further recovery possible. This restriction implies that the 'suffering' time τ (time needed to find a positive $V(x)$) cannot be arbitrarily large; this upper limit τ is set by inverting $\rho(\tau) \sim \varepsilon$, i.e.

$$\tau \sim (\ln \varepsilon)/\alpha. \qquad (5)$$

From (4) we obtain the critical value V_c

$$V_c^2 \sim \ln|\ln \varepsilon|. \qquad (6)$$

V_0 larger than V_c would imply that the species would most likely become extinct; an environment with such a V_0 value would be judged uninhabitable. On the other hand, for V_0 smaller than V_c the species would initially decrease while it is migrating to find a food shelter, then a recovery would eventually result before the population reaches the fatal lower cutoff ε.

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